Calculation of sp. heats C_p & C_v and the Ratio $\gamma = C_p/C_v$:—

Let us consider 1 mole of a gas whose molecule has n degrees of freedom each. The energy per degrees of freedom is $\frac{1}{2} kT$.

Hence energy per molecule is $\frac{1}{2} nkT$

The total energy of one mole of an ideal gas is

$$\therefore E = N\left(\frac{1}{2} knT\right)$$

where, N = Avogadro's number

$$\therefore E = \frac{1}{2} nRT$$

where $NK = R$ = gas const.

The sp. heat at constant volume

$$C_v = \frac{\partial E}{\partial T} = \frac{1}{2} nR$$

we have from the relation $C_p - C_v = R$

or, $C_p = R + C_v$

or, $C_p = R + \frac{1}{2} n R$

$\therefore \quad \gamma = \frac{C_p}{C_v}$

$$= \frac{R + \frac{1}{2} n R}{\frac{1}{2} n R}$$

$$\boxed{\gamma = 1 + \frac{2}{n}}$$

Special Case :-

For Monoatomic gases, $n = 3$ (Three degrees of freedom)

$\gamma = 1 + \frac{2}{3} = \frac{5}{3} = 1.66$

(Xe, Ne, Kr)

For diatomic gases, $n = 5$ (five degree of freedom)

$\gamma = 1 + \frac{2}{5} = \frac{7}{5} = 1.4$ (H_2, N_2, O_2)

For Triatomic gases, $n = 6$ (six degree of freedom).

$\gamma = 1 + \frac{\cancel{2}}{\cancel{6}\,3} = \frac{4}{3} = 1.33$

(SO_2, CO_2)

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The End.